Math 53: Multivariable Calculus **Sections 201, 204**

Worksheet for 2021-12-01

Warm-up

Question 1. Evaluate the integral $\int_0^1 \int_0^1 e^{\max(x^2, y^2)}\, \mathrm{d}x\, \mathrm{d}y$ where $\max(x, y)$ means the larger of x, y at each point.

Question 2. Call a parametric curve $\mathbf{r}(t)$ a *flow curve* of a vector field $\mathbf{F}$ if, at all times t, we have

$$\mathbf{r}'(t) = \mathbf{F}(\mathbf{r}(t)).$$

In other words, the velocity vector is always equal to $\mathbf{F}$ at all points along the curve.

(a) Find a flow curve of $\mathbf{F} = \langle -y, x\rangle$ that passes through the point $(1, 1)$. Hint: what does this vector field look like?

(b) Is it possible for a flow curve of a vector field to intersect itself / form a loop? What if the vector field is conservative?

Computations

Problem 1. Let $\mathbf{F} = \langle a, b, c\rangle$ where a, b, c are constants. Let D be the region $x^2 + y^2 + z^2 \leq 1$ and let E be the solid cube $-2 \leq x, y, z \leq 2$.

(a) Compute $\iint_{\partial D} \mathbf{F} \cdot \mathbf{n}\, \mathrm{d}S$ and $\iint_{\partial E} \mathbf{F} \cdot \mathbf{n}\, \mathrm{d}S$.

(b) Compute $\iint_{\partial D} |\mathbf{F} \cdot \mathbf{n}|\, \mathrm{d}S$ and $\iint_{\partial E} |\mathbf{F} \cdot \mathbf{n}|\, \mathrm{d}S$.

As usual, we orient ∂D and ∂E outwards.

Problem 2. Let $\mathbf{r}(t)$ parametrize the space curve C, with $|\mathbf{r}'(t)| = 1$ for all t.

(a) If $f(x, y, z)$ is a function on $\mathbb{R}^3$, show that

$$\frac{\mathrm{d}}{\mathrm{d}t}(f(\mathbf{r}(t))) = D_{\mathbf{r}'(t)} f.$$

(b) Suppose that $\mathbf{r}(t)$ parametrizes the curve of intersection of $2z = x + y + 3$ and $z^2 = x^2 + y^2$ with $\mathbf{r}(0) = (3, 4, 5)$ and $|\mathbf{r}'(0)| = 1$, counterclockwise when viewed from above. Find $\mathrm{d}z/\,\mathrm{d}t$ at $t = 0$. (You don't really need the previous part to do this.)